Entrainment: Local and non-local turbulence models with double diffusion

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[1] Taylor's *entrainment equation* contains the entrainment function E that has traditionally been treated heuristically, as attested by 30 different expressions for E(Ri) available in the literature. Using a model independent procedure, we first derive the new relation: $E = 2P_s h\, \overline{u}^{-3}$ which expresses E in terms of P_s, the shear production (of turbulent kinetic energy) averaged across the interface of the gravity current whose thickness and mean velocity are denoted by h and $\overline{u}$. Second, using a turbulence model for the turbulence kinetic energy K and its rate of dissipation ε (K-ε model, integrated across the flow), we compute P_s to express E in terms of the Richardson number Ri and the density ratio R_ρ characterizing double-diffusion. Third, we show that in the local (along the flow) case, the model reproduces the Ellison and Turner (1959) data while the non-local case reproduces the data by Princevac et al. (2005) which are up to ten times larger than the ET data.
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1. Introduction

[2] One of the most important processes in oceanography is represented by gravity currents whose dynamic depends critically on the rate of entrainment E of the surrounding flow [*Killworth*, 1977, 2001; *Price and Baringer*, 1994; *Gawarkiewicz and Chapman*, 1995; *Jiang and Garwood*, 1995; *Baringer and Price*, 1997a, 1997b; *Price and Yang*, 1998; *Hallberg*, 2000; *Jungclaus and Mellor*, 2000; *Astraldi et al.*, 2001; *Send and Basheck*, 2001; *Özgökmen and Chassignet*, 2002; *Girton and Sanford*, 2003; *Kase et al.*, 2003; *Özgökmen et al.*, 2003; *Wells and Wettlaufer*, 2005; *Wahlin and Cenedese*, 2005]. Traditionally, the entrainment E has been treated phenomenologically as indicated by the 30 heuristic expressions for E(Ri) cited by *Fernando* [1991]. Laboratory data by *Ellison and Turner [1959*, hereinafter referred to as ET] were represented by an empirical relation E = E(Ri) [*Turner*, 1986] which has been widely used. More recently, *Dallimore et al.* [2001] have presented an heuristic formula for E based on physical considerations of the processes that contribute to E with particular emphasis on the role of bottom friction. The conclusion is that Turner's and Dallimore et al.'s models (with a proper choice of the friction coefficient), can provide a reasonable, if not fully satisfactory, representation of a large variety of data.

[3] Quite unexpected and challenging have been the recent data by *Princevac et al.* [2005] who measured E in atmospheric (katabatic) flows. Turner's formula was found to severely underestimate E: specifically, at Ri = 0.2 and 0.5, the new E is five and ten times larger than the ET value. In addition, the new E does not vanish at Ri = 0.8, as implied by ET, but it becomes constant $\simeq$0.05 up to Ri = 1.6.

[4] Using a turbulence model, we derive an expression for E that is able to reproduce the data of ET and those of Princevac et al. We find that the first data correspond to a local regime while the latter correspond to a non-local regime.

2. Derivation of the Entrainment Equation and of E

[5] Let us denote with s and ζ the coordinates along and across a gravity current descending along the s-direction which has a slope α; the thickness of the descending plume in the ζ-direction is denoted by h [see, e.g., *Özgökmen and Chassignet*, 2002, Figure 4]. The total plume velocity **U** is decomposed into a bulk (mean) and a fluctuating part, $\mathbf{U} = \overline{\mathbf{u}} + \mathbf{u}'$. Assuming that the bulk flow if homogeneous in the y-direction, we have $\overline{\mathbf{v}} = 0$, $\partial_y A = 0$, where A represents any bulk field. In addition, one has $|\overline{\mathbf{u}}| \gg |\overline{w}|$, $|\partial_\zeta A| \gg |\partial_s A|$. Neglecting the contribution of the buoyancy force due to the variation of the plume's height, and averaging the Navier-Stokes equation over the scale of the local turbulence, the dynamic equation for the bulk velocity within the Boussinesq approximation reads:

$$\partial_t \overline{u} + \partial_s \overline{u}^2 + \partial_\zeta (\overline{u}\,\overline{w}) = -\partial_\zeta \tau_{13} + g' \sin\alpha + \nu \partial_{\zeta\zeta} \overline{u} \qquad (1a)$$

where $\tau_{ij} = \overline{u_i' u_j'}$, $g' = g\rho_p^{-1}(\rho_e - \rho_p)$ are the Reynolds stresses and the reduced gravity. Averaging (1a) over ζ via the definition [*Turner*, 1986] $\overline{A} = h^{-1}\int A d\zeta$, the last term in the lhs and the first term in the rhs of (1a) vanish. Changing the order of integration and differentiation in the remaining terms in the lhs of (1a) and employing the top-hat approximation the steady state version of (1a) gives [*Turner*, 1973, equation 6.2.6]:

$$\partial_s\left(h\overline{u}^2\right) = g' h \sin\alpha - u_*^2 \qquad (1b)$$

where $u_* = [-\nu \partial_\zeta u(0)]^{1/2}$ is the friction velocity. Next, from (1a) we derive the dynamic equation for the mean kinetic energy $\overline{K} = \overline{u}^2/2$:

$$\partial_t \overline{K} + \partial_s\left(\overline{u}\overline{K}\right) + \partial_\zeta\left(\overline{w}\overline{K}\right) = -\overline{u}\partial_\zeta \tau_{13} + \overline{u} g' \sin\alpha + \nu \overline{u} \partial_{\zeta\zeta} \overline{u} \qquad (1c)$$

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To average (1c), consider the first term in the rhs of (1c). We divide the region of integration into two parts separated by the point ζ_0 where $\partial_\zeta \bar{u}(\zeta) = 0$. In almost the entire region below ζ_0, we have $\tau_{13} = -u_*^2$ while above ζ_0 the function $\tau_{13}(\zeta)$ changes sign and thus $\tau_{13}(\zeta_0) = 0$. It follows that:

$$\int_0^{\zeta_0} d\zeta \bar{u} \partial_\zeta \tau_{13} = -\int_0^{\zeta_0} d\zeta \tau_{13} \partial_\zeta \bar{u} = \bar{u} u_*^2 \tag{1d}$$

while the integration over the upper part yields:

$$\int_{\zeta_0}^{\infty} d\zeta \bar{u} \partial_\zeta \tau_{13} = \int_{\zeta_0}^{\infty} d\zeta \, P_s(\zeta) \equiv hP_s \tag{1e}$$

where P_s is the shear interface production. Assuming a steady state, averaging (1c) in the top-hat approximation and neglecting the viscous terms which do not greatly affect the large scale flow, we obtain:

$$\partial_s\left(\frac{1}{2} h\bar{u}^3\right) = -hP_s + \bar{u}hg' \sin\alpha - \bar{u}u_*^2 \tag{1f}$$

Eliminating g' between (1f) and (1b), we obtain:

$$\partial_s(h\bar{u}) = E\bar{u}, \qquad E = 2hP_s\bar{u}^{-3} \tag{1g}$$

We notice that $u_*^2 = C_D\bar{u}^2$ has canceled out in relations (1g). This is due to the fact that in integrating the Reynolds stresses, we assumed that below the inversion point of the mean velocity profile (defined as the point where $\partial_\zeta \bar{u}(\zeta_0) = 0$, see Figure 8b of Dallimore et al. and Figure 3 of Princevac et al.), we have (1d) which then cancels the analogous term in equation (1b). Even though the production due to bottom friction is larger than the one in the region $\zeta > \zeta_0$ just considered (Figure 16 of Dallimore et al.), in the present formulation such a term cancels out, a conclusion that can be directly checked. Using Figure 16 of Dallimore et al., the production in the $\zeta > \zeta_0$ region can be computed to be $hP_s = 7*10^{-7}$ m^3 s^{-3} and using their Table 1 the mean velocity turns out to be $\bar{u} = 0.21$ ms^{-1}. Inserting these values into the second of (1g), the present model predicts that $E = 1.5*10^{-4}$ which compares well with the value $E = 1.4*10^{-4}$ of *Dallimore et al.* [2001], thus confirming the cancellation of the friction terms, and consequently the validity of equations (1d, 1g).

[6] In conclusion, we have derived an expression for E which is model independent since no turbulence model was used. Below we show how E depends on the large scale parameters Ri and R_ρ.

3. Computation of E Versus Ri and R_ρ

[7] The Reynolds Stress Model (RSM [*Canuto et al.*, 2001, hereinafter referred to as I; *Canuto et al.*, 2002, hereinafter referred to as II]) yields the following results for P_s, the rate of production of turbulent kinetic energy by shear:

$$P_s = K_m\Sigma^2 \qquad K_\alpha = 2K^2\varepsilon^{-1}S_\alpha(Ri, R_\rho) \tag{2a}$$

where K_α is the diffusivity of an arbitrary field (α = momentum, heat, salt), K is the turbulent kinetic energy, ε its rate of dissipation while the S_α's are dimensionless structure functions of Ri and R_ρ discussed and plotted in I-II (see section 5). Furthermore, $Ri = N^2\Sigma^{-2}$ is the Richardson number, N ($N^2 = -g\rho_0^{-1}\partial\rho/\partial z$) is the Brunt-Väisäla frequency, Σ is the mean shear and R_ρ is the density ratio $R_\rho = (\beta\partial S/\partial z)(\alpha\partial T/\partial z)^{-1}$. To evaluate P_s, one needs K and ε for which we adopt the K-ε model ($D/Dt = \partial_t + \mathbf{\bar{u}} \bullet \nabla$; $\tau = 2K/\varepsilon$; $c_{1,2}$ = 2.88, 3.8):

$$D_tK + \partial_i\overline{Ku_i} = P - \varepsilon \tag{2b}$$

$$D_t\varepsilon + \partial_i\overline{\varepsilon u_i} = c_1\tau^{-1}P - c_2\varepsilon\tau^{-1} \tag{2c}$$

where P is the total production due to shear, temperature and salinity given by (I, II):

$$P = P_s + P_T + P_S = K_m\Sigma^2 - K_\rho N^2 = P_s\left(1 - RiS_\rho S_m^{-1}\right) \equiv P_s\varphi \tag{2d}$$

and K_ρ is the "mass diffusivity" given in terms of the heat-salt diffusivities $K_{h,s}$:

$$K_\rho = \left(K_h - R_\rho K_s\right)\left(1 - R_\rho\right)^{-1} \tag{2e}$$

When we average equations (2b, 2c) across the flow, the terms ∂_ζ give zero contribution while for the along-the-flow non-local terms, we suggest the closures:

$$\partial_s(h\overline{\varepsilon u}) = c_\varepsilon\varepsilon K^{1/2}, \quad \partial_s\left(h\overline{uK}\right) = 2^{1/2}aK^{3/2}, \quad \bar{u}\partial_s(hK) = b\bar{u}K \tag{2f}$$

where for simplicity we have used the notation $K = h^{-1}\int K(\zeta)d\zeta$, $\varepsilon = h^{-1}\int\varepsilon(\zeta)d\zeta$ and where the coefficients a, b, c_ε will be discussed later (section 7). Using the variables $\psi^2 \equiv h\varepsilon\bar{u}^{-3}$ and $\tau = 2K/\varepsilon$, we then have:

$$\varepsilon^{-1}\tau h^{-1}\partial_s(h\overline{\varepsilon u}) = 2^{-1/2}c_\varepsilon\psi(\tau\Sigma)^{3/2} \tag{2g}$$

$$x \equiv \varepsilon^{-1}h^{-1}\left[\partial_s\left(h\overline{uK}\right) + \bar{u}\partial_s(hK)\right] = \frac{1}{2}a\psi(\tau\Sigma)^{3/2} + \frac{1}{2}b(\tau\Sigma) \tag{2h}$$

The stationary solution of the K-ε model, together with (1g) and (2g, 2h), then yields:

$$E = (\tau\Sigma)^2 S_m\psi^2, \quad K = \frac{1}{2}\bar{u}^2\psi^2\tau\Sigma, \quad \psi^2 \equiv h\varepsilon\,\bar{u}^{-3}, \quad K_\alpha = \frac{1}{2}\bar{u}hE \tag{3a}$$

$$\begin{aligned} \frac{1}{2}(\tau\Sigma)^2 &= (1 + x)(\varphi S_m)^{-1}, \\ 2^{1/2}\psi &= c_1c_\varepsilon^{-1}(\tau\Sigma)^{1/2}\left[\varphi S_m - 2c_2c_1^{-1}(\tau\Sigma)^{-2}\right] \end{aligned} \tag{3b}$$

To solve equation (3b), we need the structure functions $S_{m,h}(Ri, R_\rho, \tau\Sigma)$ which are discussed in section 5. The procedure is therefore as follows. Using (2h) in (3b) with the $S_{m,h}(Ri, R_\rho, \tau\Sigma)$ discussed below, one solves for $\tau\Sigma$ vs. (Ri, R_ρ) which is then used in the first of (3a) to obtain the

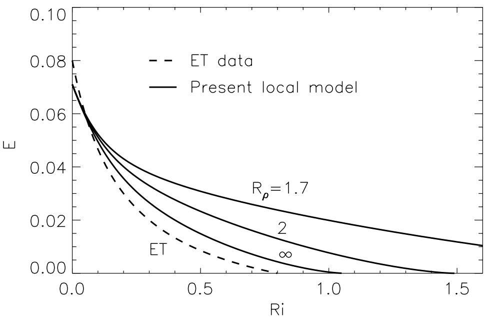

Figure 1. The entrainment function E vs. Ri for the local case. The dashed line corresponds to the Turner's model, equation 4c. The solid curves correspond to different values of the density ratio. Without double diffusion, corresponding to $R_\rho = 0$, (or ∞), the function E is very close to the Turner's curve.

desired relation $E(Ri, R_\rho)$. Finally, the local-case $P = \varepsilon$ corresponds to $x = 0$ while the non-local case corresponds to $x \neq 0$.

4. Double Diffusion

[8] To account for DD processes, one must use the form of $S_{m,h}(Ri, R_\rho, \tau\Sigma)$ given by equations (13–15) of II. At high shear values, corresponding to small Ri, many data have shown [e.g., *Linden*, 1971] that heat and salt diffusivities are the same and thus it follows from (2e) the mass diffusivity is the same as that of heat and salt. The resulting E must therefore be essentially the same as in the case without DD. At higher Ri, when shear subsides, mixing weakens and DD processes may play a role. We exhibit this behavior in the local case, Figure 1.

5. The Structure Functions S_α

[9] There is a long and interesting history about the dimensionless structure functions $S_\alpha(Ri, R_\rho, \tau\Sigma)$ which that dates back to the original Mellor-Yamada (MY) model and which has been recently discussed in detail [*Cheng et al.*, 2002, section 6]. First, no model included the dependence on R_ρ until model II. Second, all models exhibited an inverse dependence of S_m on Ri, as expected in stably stratified flows. However, the MY-type models cut off mixing too early leading to shallow mixed layers [*Martin*, 1985]. The turbulence model developed in II (see Figures 4 and 5 of II) included a more physical representation of several aspects of the RSM, in particular a better representation of the velocity and temperature pressure correlations. In turn, that implied a S_α vs. Ri relationship that allowed mixing to exist much longer, thus yielding more realistic mixed layer depths [*Burchard and Bolding*, 2001]. In addition, *Burchard and Deleersnijder* [2001] showed that the new S_α are more stable and more well behaved than previous ones. In the context of DD processes, one expects that under the transformation $R_\rho \rightarrow 1/R_\rho$, the structure functions S_h and S_s exchange their roles. It was verified that this is indeed the case. The full expression for the functions S_α (Ri, R_ρ, $\tau\Sigma$) is given in equations 13–15 of II.

[10] Since the general effect of DD processes on the entrainment E is exhibited in Figure 1 for the local case, in the more complex non-local case we turn off the DD processes contribution. In that case, the structure function $S_m(Ri, \tau\Sigma)$ is given by equations (17–18) of *Cheng et al.* [2002]:

$$S_m(Ri, \tau\Sigma) = [s_0 + (s_1 Ri + s_2)\sigma] \cdot \left[1 + (d_1 Ri + d_2)\sigma + \left(d_5 + d_4 Ri + d_3 Ri^2\right)\sigma^2\right]^{-1} \tag{3c}$$

where $\sigma \equiv (\tau\Sigma)^2$. The constants s_n and d_n are given in Table 2 of *Cheng et al.* [2002].

6. Bottom Friction

[11] Although we have presented arguments to justify the cancellation of bottom friction terms in (1g), one cannot exclude a residual contribution as a source of turbulent kinetic energy of the form:

$$P_* = C_* u_*^3 h^{-1} \qquad C_* \approx 1 \tag{3d}$$

which would change the total production (2d) to:

$$P = P_s\varphi + C_* C_D^{3/2} h^{-1} \bar{u}^3 = P_s\varphi\left(1 + pE^{-1}\right), \; p = 2C_* C_D^{3/2}\varphi^{-1} \tag{3e}$$

which leads to a change in the previous expressions of φ into $\varphi(1 + pE^{-1})$. It follows that bottom friction plays a role when p is not too small which occurs when:

$$C_D^{3/2} \leq E \tag{3f}$$

With typical values of $C_D = (3–5)*10^{-3}$, we obtain $E = (1.5–3.5)*10^{-4}$. We conclude that bottom friction contributes to the entrainment in the oceanic regions characterized by Froude's number of order unity, as one can see from the plot of E vs Fr in Figure 8 of *Wells and Wettlaufer* [2005] and in Figure 2 of *Wahlin and Cenedese* [2005].

7. Local Versus Non-Local Models

[12] A local (along the flow) model corresponds to $a = b = 0$ in equation (2h) and thus $x = 0$ in which limit, equation (3b) gives:

$$(\tau\Sigma)^2 S_m = 2\varphi^{-1} \qquad \psi^2 \sim \varphi^{3/2} S_m^{3/2} \tag{4a}$$

equations (3a) and (2a) imply that the entrainment E, the turbulent kinetic energy K, the dissipation ε and the momentum diffusivity K_m are then given by:

$$E \sim S_m^{3/2}, \; K \sim \frac{1}{2}\bar{u}^2 S_m, \; \varepsilon \sim h^{-1}\bar{u}^3 S_m^{3/2}, \; K_m \sim h\bar{u} S_m^{3/2} \tag{4b}$$

Since S_m vanish as Ri approaches unity, all the variables in (4b) vanish as $Ri \rightarrow 1$, as shown in Figure 1 where we also plot Turner's relation:

$$E(Ri) = (0.08 - 0.1Ri)(1 + 5Ri)^{-1} \tag{4c}$$

Dallimore et al. [2001] have suggested a relation E(Ri) that depends on the drag coefficient C_D and in their Figure 10

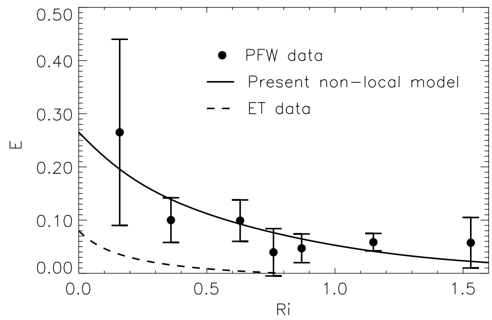

Figure 2. The entrainment function E vs. Ri for the non-local case (a = b = -0.035, c_ε = -0.35). The data by *Princevac et al.* [2005] are presented with the error bars. The model results correspond to the solid line. The dashed line is the Turner's expression which is added to highlight the much lower values of the local case.

the authors present a variety of data of E vs.Ri for which their relation gives a reasonable fit with $C_D = (5 - 15)*10^{-3}$. Some considerations are in order. equation (4c) depends only on Ri since the experiment by Ellison and Turner were carried out at R_ρ = 0. It implies that when Ri > 0.8, the entrainment E vanishes. The new model suggests that when shear mixing becomes negligible at Ri = O(1), either salt-fingering ($0.7 \le R_\rho \le 1$) and/or diffusive convection (R_ρ > 1) may become operative and yield a non-zero entrainment. Whether this prediction is correct or not remains to be seen (when shear is large corresponding to a small Ri, DD processes are not expected to be important and in fact the curves for different R_ρ become indistinguishable from the ET case, Figure 1). Several other empirical relations of E vs.Ri have been proposed since 1969 and a list of 30 of them is given by *Fernando* [1991]. As one can see, our local model reproduces well the Ellison-Turner data.

[13] In the *Princevac et al.* [2005] experimental set up, the breaking action by entrainment and drag is much larger than the accelerating force due to reduced buoyancy (the g' term in 1b) and this implies a decrease of all turbulent properties along the flow's motion. For this reason, we took all ∂_s in (2b, 2c) to be negative and as a result, the coefficients a,b,c_ε in equations (2g, 2h) were taken to be negative. In Figure 2 we present the non-local model results (without double diffusion). The *Princevac et al.* [2005] data, with the corresponding error bars, are reproduced rather well. To highlight the large difference with the new data, we have added the ET data (dotted line).

8. Conclusions

[14] Although it has long been recognized that the instabilities occurring at the plume's interface is the physical cause of local mixing leading to entrainment, we believe that this work provides the first model independent relation between E and the rate of interface shear production, equation (1g).

[15] Using K-ε turbulence model (averaged across the flow), we computed E in both local and non-local cases (in the along the flow direction). We show that the ET data (*Turner*'s [1986] relation) are well reproduced by the local model, while the new data by *Princevac et al.* [2005] are reproduced by the non-local model. Thus, the same model can explain data that are quite different since the Princevac et al. data yield an entrainment E which is up to an order of magnitude larger than the ET data.

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